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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

198. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve $2^{x+y}=6^y$; $3^x=3.2^{y+1}$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and A. H. HOLMES, Brunswick, Me.

$2^{x+y}=6^y$, gives $2^x=3^y$..............(1), $3^x=3.2^{y+1}$, gives $3^{x-1}=2^{y+1}$..............(2).
(1) multiplied by (2) gives $6^x=6^{y+1}$..............(3).
From (1), $x/y=\log 3/\log 2$; from (3), $x=y+1$.
$\therefore y=\log 2/(\log 3-\log 2)$, $x=\log 3/(\log 3-\log 2)$.

Also solved by F. D. Posey, J. E. Sanders, G. W. Greenwood, Christian Hornung, L. E. Newcomb, J. Scheffer, Grace M. Bareis, and the Proposer.

199. Proposed by SAUL EPSTEEN, Ph. D., Chicago, Ill.

Solve $(x-a_1)(x-a_2)(x-a_3)(x-a_4)(x-a_6)=(x+a_1)(x+a_2)(x+a_3)(x+a_4)(x+a_5)(x+a_6)$.

Solution by F. D. POSEY, A. B., San Mateo, Cal., and G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let $p_1=\Sigma a_1$, $p_2=\Sigma a_1a_2$, etc. Expanding both members of the equation we have $x^6-p_1x^5+p_2x^4-p_3x^3+p_4x^2-p_5x+p_6=x^6+p_1x^5+p_2x^4+p_3x^3+p_4x^2+p_5x+p_6$, which reduces to $p_1x^5+p_3x^3+p_5x=0$. One root is therefore 0. Dividing by x, we have $p_1x^4+p_3x^2+p_5=0$.

Solving the latter as a quadratic we have $x^2=[-p_3\pm(p_3^2-4p_1p_5)^{\frac{1}{2}}]/2p_1$.

$$\therefore x=\pm\left[\frac{-p_3\pm(p_3^2-4p_1p_5)^{\frac{1}{2}}}{2p_1}\right]^{\frac{1}{2}}.$$

Since the original equation is of the sixth degree, one root is infinite.

Also solved by G. W. Greenwood, B. A. (Oxon), and L. E. Newcomb.

GEOMETRY.

220. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Two triangles are circumscribed to a given triangle ABC, having their sides perpendicular to the sides of the given triangle. Prove that the two triangles are equal, and find the area of these triangles.

III. Solution by J. SCHEFFER, Kee Mar College, Hagerstown, Md.

Let $A'B'=c'$ be the side of the circumscribed triangle drawn through B and perpendicular to AB, $A'C'=b'$ through A and perpendicular to AC, and $B'C'=a'$ through C and perpendicular to BC. It is seen at once that triangle $A'B'C'$ is equiangular with triangle ABC. We have

$$A'B'=c'=c\cot A+\frac{a}{\sin B}=\frac{c\cos A}{\sin A}+\frac{c\sin A}{\sin B\sin C}=c\,\frac{\cos A\sin B\sin C+\sin^2 A}{\sin A\sin B\sin C}$$

$$=e\,\frac{1-\cos A(\cos A-\sin B\sin C)}{\sin A\sin B\sin C}=c\,\frac{1+\cos A\cos B\cos C}{\sin A\sin B\sin C}$$

$$=c(\operatorname{cosec}A\operatorname{cosec}B\operatorname{cosec}C+\cot A\cot B\cot C)=c(\cot A+\cot B+\cot C).$$

By analogy $b'=b(\cot A+\cot B+\cot C)$, $a'=a(\cot A+\cot B+\cot C)$. This proves the equality of all circumscribed triangles, the ratio $(\cot A+\cot B+\cot C)$ being the same. Denoting the triangle ABC by $\triangle$, this ratio may also be expressed $\frac{a^2+b^2+c^2}{4\triangle}$. Denoting the area of the circumscribed triangle by $\triangle'$, we have evidently $\triangle'=(\cot A+\cot B+\cot C)^2\triangle=\frac{(a^2+b^2+c^2)^2}{16\triangle}$.

222. Proposed by **G. B. M. ZERR, A. M., Ph. D.,** Parsons, W. Va.

At the ends of a focal chord CC' of a parabola are drawn the normal chords CD, $C'D'$. Prove that DD' is parallel to CC' and equal to three times its length.

III. Solution by **J. SCHEFFER,** Kee Mar College, Hagerstown, Md.

It is a well-known theorem that the two tangents at the extremities of any focal chord in a parabola intersect at right angles on the directrix. Choosing two such tangents for the axes of coördinates, the lengths of the tangents between the origin and points of contact being a and b, we have $(x/a+y/b-1)^2=4xy/ab$ as the equation of the parabola, and $x/a+y/b=1$ the equation of the focal chord. The equation of the normal at C' is $x=a$, and at C, $y=b$. Substituting these in the equation of the parabola we get for the point D, $(a, 4b)$ and for D', $(4a, b)$. Consequently the equation for DD' is $y=-(b/a)x+3b$, which shows that it is parallel to CC'. $DD'=\sqrt{(9a^2+9b^2)}=3\sqrt{(a^2+b^2)}$; and $CC'=\sqrt{(a^2+b^2)}$, which proves $DD'=3CC'$.

223. Also solved by **L. E. Newcomb,** Los Gatos, Cal.

224. Proposed by **WILLIAM HOOVER, Ph. D.,** Professor of Mathematics, Ohio State University, Athens, O.

The equations to two circles are $(x-a)^2+(y-b)^2=c^2$, $(x-b)^2+(y-a)^2=c^2$; give the length of their common tangent and *thence* the condition that the two circles may touch.

Solution by **J. SCHEFFER,** Hagerstown, Md.; **F. D. POSEY, A. B.,** San Mateo, Cal.; **G. B. M. ZERR, A. M., Ph. D.,** Parsons, W. Va.; **A. H. HOLMES,** Brunswick, Me.; **J. E. SANDERS,** Hackney, O.; **CHRISTIAN HORNUNG, A. M.,** Tiffin, O.; **E. L. SHERWOOD,** Pittsburg, Pa.; **GRACE M. BAREIS,** Bala, Pa.

Since the circles are equal the length of a common external tangent is equal to the distance between the centers which is $\sqrt{[(a-b)^2+(b-a)^2]}=(a-b)\sqrt{2}$.

$\therefore$ The circles will touch if $(a-b)^2=2c^2$.

Also solved by **L. E. Newcomb,** Los Gatos, Cal.